Electromagnetic Scattering of Finite Strip Array on a Dielectric Slab

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***Abstract*—A fast recursive algorithm is used to compute the scattering properties of finite array of strip gratings on a dielectric slab. This algorithm has a computational complexity of $O(N\log^2 N)$ for one incident angle and $O(N^2\log N)$ for N incident angles. It uses plane wave basis for expanding the incident wave and the scattered wave. The scattered wave is expanded in terms of a Sommerfeld-type integral with spectral distribution along a vertical branch cut, rendering its expansion very efficient. To validate the scattering solution obtained using the recursive algorithm, comparisons with the method of moments are illustrated. The current distributions on the strips and scattering patterns are both presented. Since this algorithm has reduced computational complexity and is fast compared to other conventional methods, it can be used to analyze very large strip arrays. Scattering solution of a 50-wavelength wide strip is illustrated.**

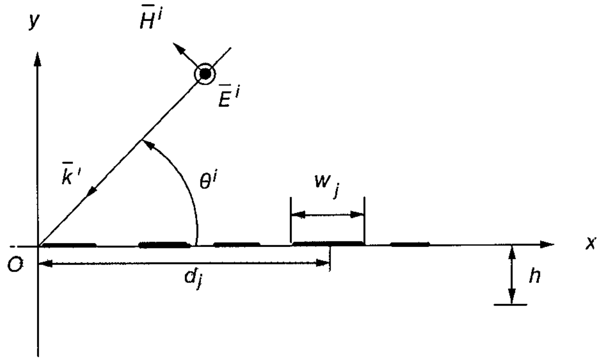

Fig. 1. Geometry of the problem.

I. Introduction

THE WAVE scattering by strip gratings supported by a dielectric slab remains a canonical problem in wave scattering theory. It also finds applications in a number of areas like frequency selective surface, slow-wave structures, leaky-wave antennas, and microstrip arrays. For instance, in frequency-selective-surface applications, it could be used to reduce radar cross sections. Hence, wave scattering from strip gratings has been investigated by many authors [1]–[6]. However, most works are limited to infinite periodic gratings.

Recently, we have developed a fast recursive method for dielectric scattering solutions [7]. This method has been adapted to deriving the solution of a finite nonperiodical strip arrays or a single strip [8] suspended in free space. However, for practical applications, the strip arrays are usually supported by a dielectric slab. This paper extends the method of the previous work [8] to strip arrays supported by a dielectric slab. Due to the proximity of the slab to the strip array, the introduction of the slab will change the scattering properties of the strip array. This change is caused by the interaction between strips and the slab. Numerical results indicate that this interaction is apparent by studying the current distribution of a large strip.

In this implementation of the recursive algorithm, plane waves are used as bases for the incident wave as well as the scattered waves. In particular, the scattered wave is expanded with spectral distribution along a vertical branch cut of the Sommerfeld-type integral. This greatly expedites the representation of the waves, and results in a computational algorithm with $O(N\log^2 N)$ complexity for a single incident wave, and with $O(N^2\log N)$ complexity for N incident waves.

II. Formulation

Fig. 1 shows the geometry of strips which are infinitely thin perfect conductors supported by a dielectric slab. The relative dielectric constant of the slab is ϵ_r. The ith strip is located at d_i, with width w_i, where $i = 1, 2, \cdots, N$, and N is the total number of the strips. For a uniform array, we use a to denote the width of each strip and d, the distance between the centers of two adjacent strips. When a plane wave is incident, it will induce current (and charge) distributions on the strips, which in turn radiate to form the scattered waves. Here, the induced current (and charge) is unknown. It can be solved for by the recursive method illustrated in detail in [8]. In the following, we shall first derive the Green's function for this geometry, and then illustrate how the recursive method can be applied to this problem. In the following analysis, the time dependence of $e^{-i\omega t}$ is suppressed throughout and $(\partial/\partial z) = 0$.

Using the spectral technique, for H_z polarized field, the x-component of the electric field due to an x- directed line current source is given by [10]

$$E_x = i\omega\mu_0\left(1 + \frac{1}{k^2}\frac{\partial^2}{\partial x^2}\right)A_x, \tag{1}$$

where

$$A_x = \frac{i}{4\pi}\int_{-\infty}^{\infty}\frac{1}{k_y}(e^{ik_y|y|} - R^{\mathrm{TM}}e^{ik_y y})e^{ik_x(x-x')}\,dk_x. \tag{1a}$$

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In the same way, we can derive the corresponding field for E_z polarized case. It is

$$A_z = \frac{i}{4\pi}\int_{-\infty}^{\infty}\frac{1}{k_y}(e^{ik_y|y|} + R^{\mathrm{TE}}e^{ik_y y})e^{ik_x(x-x')}\,dk_x, \quad (2)$$

$$E_z = i\omega\mu_0 A_z. \quad (2a)$$

In the above, R^{TM} and R^{TE} are generalized reflection coefficients [10] for TM-to-y and TE-to-y waves, respectively. They have different forms for different configurations.

In summary, the fields due to a line source in a planarly layered medium can be expressed in terms of one component of the potential A. When the source is on the boundary ($y' = 0$) and for matching boundary condition, we need only consider the field at $y = 0$. In this case, A has the following form

$$A(x-x') = \frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{dk_x}{k_y}(1 \pm R)e^{ik_x(x-x')}, \quad (3)$$

where, $k_y^2 = k^2 - k_x^2$. The "+" sign corresponds to TE case with $A = A_x$ and $R = R^{\mathrm{TE}}$, whereas the "−" sign corresponds to TM case with $A = A_z$ and $R = R^{\mathrm{TM}}$ [10].

The inversion path of the Fourier integral in (3) is taken to be the Sommerfeld integration path, which is slightly above the real axis for $k_x < 0$ and slightly below the real axis for $k_x > 0$ [10]. When $x - x'$ is large, the integrand becomes a rapidly oscillating function of k_x, and its numerical evaluation becomes difficult. To expedite the numerical evaluation of the integral, the path of the integration is deformed to a branch cut. Hence, when h is small, (3) is equal to a branch cut integral plus the contribution from the pole of R at $k_x = k_{xp}$, i.e.,

$$A(x-x') = -\frac{1}{4\pi}\int_0^{\infty}\frac{2 + R(k_x,k_y) + R(k_x,-k_y)}{k_y}e^{(ik-s)|x-x'|}\,ds + f_p e^{ik_{xp}|x-x'|}, \quad (4)$$

where

$$f_p = 2\pi i \lim_{k_x \to k_{xp}}(k_x - k_{xp})R(k_x,k_y). \quad (4a)$$

The branch-cut integral corresponds to a space wave with the phase velocity of free space. It corresponds to a wave with most of its energy travelling in the free space. The pole corresponds to a guided mode in the slab region [10], [11]. Since it is a guided mode with phase velocity slower than that of the free space, it will be evanescent in the free-space region. Hence, it is often called a surface wave. Also, when the slab thickness is small, the guided wave has a phase velocity close to that of free space [11].

After the integral is truncated and discretized, (4) can be expressed as

$$A(x-x') = \sum_{m=1}^{M+1} g_m e^{-u_m|x-x'|}, \quad (5)$$

where $u_m = s_m - ik, s_m$ and $g_m, m = 1,2,\cdots,M$ are numerical integration points and coefficients, respectively, $u_{M+1} = -ik_{xp}$, and $g_{M+1} = f_p$.

Equation (5) indicates that the scattered field from a strip can be expanded in terms of a plane-wave basis. Consequently, this allows a T matrix be defined for a single substrip using a plane wave basis. Then, the recursive algorithm described in [7] and [10] can be used to find the scattering solution of a finite strip array. The adapted algorithm is described in [8].

Formally, (5) expands the field using the plane wave basis which is similar to the method used in [8]. Hence the recursive algorithm described in [8] can be used for this problem without much change. Again, in order to reduce the computational complexity, the discrete points $s_m, m = 1,2,\cdots,M$, are unevenly distributed which is the densest at $s = 0$ and becomes increasingly coarser when $s \to \infty$. In this manner, the number of sampling needed for the integral is minimized, and grows only logarithmically with the size of the problem.

To test the validity of the recursive result, method of moments is also used to solve the same problem. In this method, the Green's function is found by evaluating the Sommerfeld integral numerically. Since the convergence of the integration in (4) is very slow, we rewrite it into the following form to accelerate its convergence:

$$A(x-x') = \frac{i}{4}(1 + R_\infty)H_0^{(1)}(k|x-x'|) + \frac{i}{2\pi}\int_0^{\infty}\frac{R - R_\infty}{k_y}\cos k_x(x-x')\,dk_x, \quad (6)$$

where

$$R_\infty = \lim_{k_x\to\infty}R(k_x,k_y) = (1-\epsilon_r)/(1+\epsilon_r). \quad (6a)$$

We can show that

$$R - R_\infty \propto 1/k_x^2, (k_x \to \infty). \quad (7)$$

Hence, the convergence of (6) is much faster than that of (4).

III. Computational Complexity

The computational complexity of the above algorithm can be analyzed as before [7]–[10]. Previous analysis shows the complexity of the recursive algorithm to be $O(NP^2)$, where P is the number of harmonics needed to represent the aggregate T matrix. The size of the aggregate T matrix is $P \times P$.

In this problem, since the plane waves are expanded on a branch-cut integral which are efficient, and that the number of sampling points grows logarithmically as the size of the problem, the number of plane waves P needed to represent the scattered field or the aggregate T matrix is proportional to $\log N$. Hence, the total complexity of the recursive algorithm is $O(N\log^2 N)$.

When N incident waves is assumed in this problem, the aggregate T matrix is no longer square, but an $(N+P)\times P$ matrix [8]. The total computational complexity of the algorithm can be shown to be $O(N(N+P)P)$ which is $O(N^2\log N)$ when N is large. Hence, this algorithm represents a fast way of solving this scattering problem with reduced complexity compared to other methods of solving this problem.

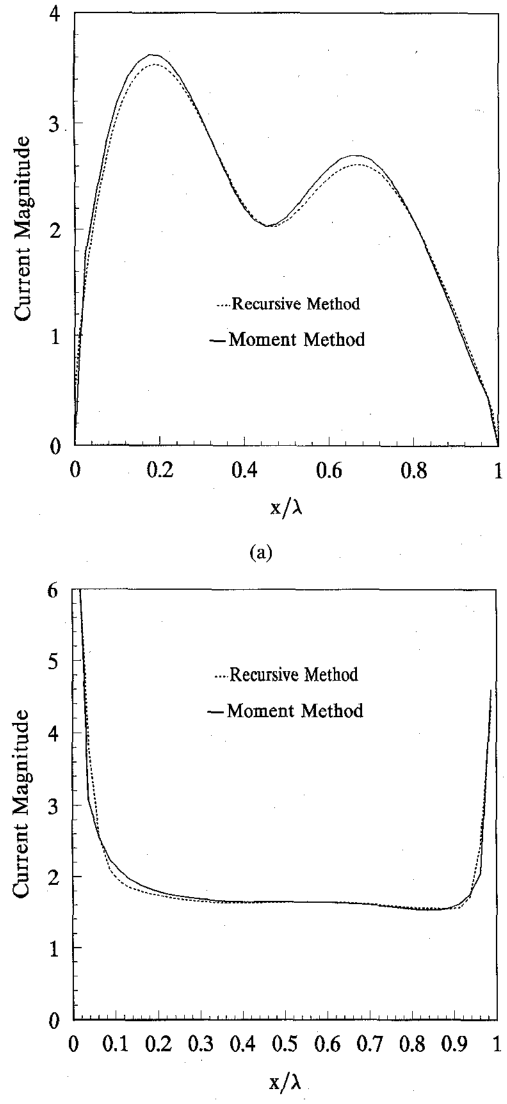

Fig. 2. Induced current of a 1λ strip on a slab without a ground plane ($\epsilon_r = 2.1, h = 0.1\lambda$). (a) H_z incidence at $\theta^i = 45°$; (b) E_z incidence at $\theta^i = 60°$.

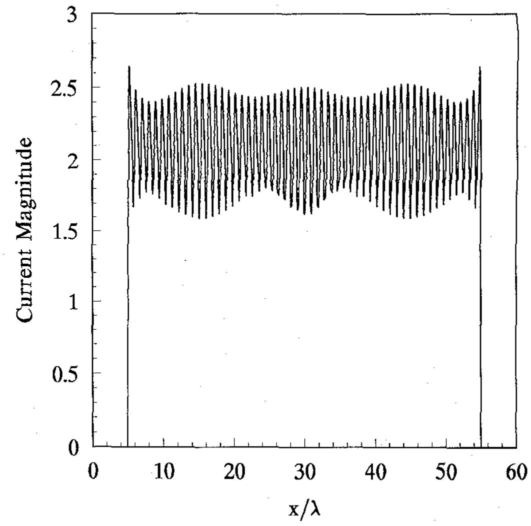

Fig. 3. Induced current of a 50λ strip on a slab without a ground plane (H_z incident wave at $\theta^i = 90°$).

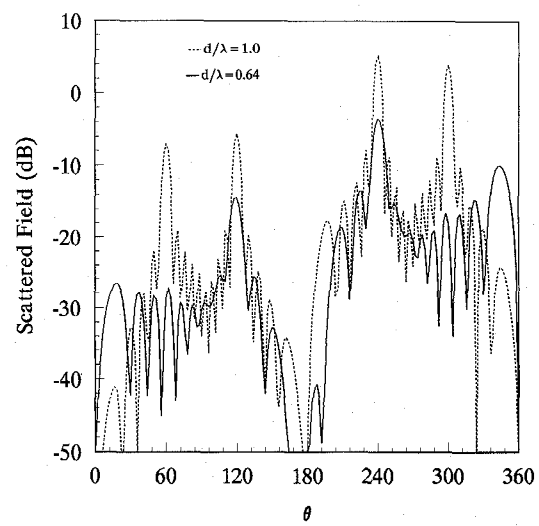

Fig. 4. Scattering pattern of a uniform array of 10 strips on a slab without a ground plane ($\epsilon_r = 2.57, h = 0.171\lambda, a = d/2, E_z$-wave incidence at $\theta^i = 60°$).

IV. Numerical Results

In order to test the validity of the algorithm and analyze the scattering properties of slab, we computed the current distribution for several single strips and compared the solution with the method of moments. Motivated by frequency-selective surface applications, the following studies will be done without the presence of the ground plane. Notice, however, that frequency-selective surfaces are often studied by assuming infinite periodicity so that Floquet theorem can be invoked. However, in practice, the frequency-selective surfaces are truncated and finite in extent. The algorithm here represents an efficient way to study finite-extent frequency-selective surfaces where Floquet theorem can not be invoked.

Fig. 2(a) shows the current of a 1λ strip on a slab ($\epsilon_r = 2.1, h = 0.1\lambda$) for H_z incident wave at $\theta^i = 45°$. The moment method result is also shown in this figure. The agreement between them is good. Fig. 2(b) is the result of the same problem for E_z polarized case with $\theta^i = 60°$.

Fig. 3 shows the current of a larger strip ($w = 50\lambda, H_z$ incident wave at $\theta^i = 90°, \epsilon_r = 2.6, h = 0.1\lambda$). From this figure, we observe the effect of the interaction between the incident wave ($e^{ik_x^i x}$) and the guided wave ($e^{i\xi_p x}$). The fast variation is due to the sum of their spatial frequencies while the slower variation is due to the difference of their spatial frequencies. This interaction was not observed when the strips were suspended in air, because there was no guided mode on the dielectric slab [8].

For a strip array, we computed the scattering pattern of a 10 strip uniform array. The result is shown in Fig. 4. We find that as the frequency changes, the reflected field also changes. To see this more clearly, we draw a curve of the RCS in the reflect direction versus normalized d, which is the separation between strips, as shown in Fig. 5. The interference property of the strips, where different magnitudes of reflections and transmissions can occur at different frequencies, is often used for frequency-selective surface applications.

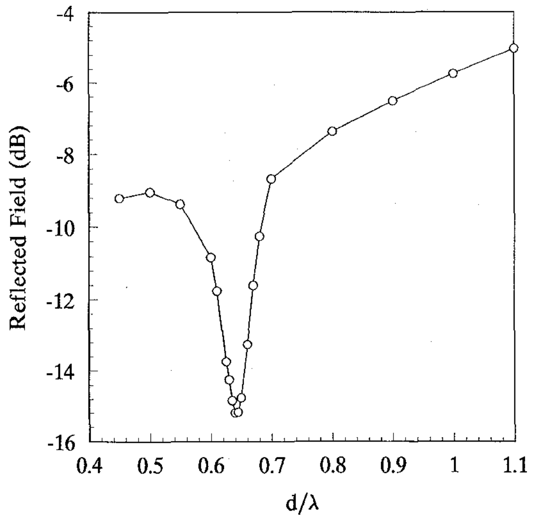

Fig. 5. RCS versus d/λ for a uniform array of 10 strips on a slab without a ground plane ($\epsilon_r = 2.57, h = 0.171\lambda, a = d/2, E_z$-wave incidence at $\theta^i = 60°$).

V. Conclusion

In conclusion, we have demonstrated a fast algorithm with reduced computation complexity for computing current distributions and scattered fields of strips and strip arrays on a dielectric slab. This algorithm is shown to work for both the benign E_z polarization and the pathological H_z polarization. Because of its reduced computational complexity, the algorithm provides a way to analyze large, finite, planar strip gratings supported by a dielectric slab. By changing the reflection coefficients in (3), this algorithm can be applied to the above grating structure with a dielectric slab on a conductor ground.

Compared to the strips without dielectric slab, we now have to account for the effect of the surface wave (or guided wave) pole in the interaction between the strips. When the slab is thin compared to wavelength, the phase velocity of the surface wave (pole contribution) is almost the same of that of the space wave (branch cut contribution). The interference between the surface wave and the space wave results in a slow variation in the current distribution on a wide strip.

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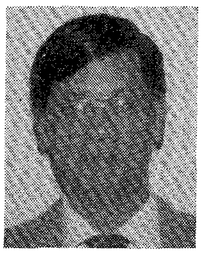

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